Turbulent Pipe Flow Velocity Profile Model for Drag-Reducing Fluids

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The phenomenon of drag reduction in turbulent flow upon the addition of minute quantities of certain additives has been the subject of extensive study due its obvious pragmatic importance in the shipbuilding industries, firefighting operations, oil well fracturing processes, and irrigation systems. A summary of the tremendous work done on drag reduction can be obtained from a number of comprehensive reviews on the subject (Lumley, 1969; Patterson et al., 1969; Gadd, 1971; Darby, 1972; Hoyt, 1972; Landahl, 1973; Lumley, 1973; Fisher and Ash, 1974; Palyvos, 1974; Virk, 1975; Shenoy, 1976; Ting, 1982; Shenoy, 1984). A number of empirical relationships for correlating and predicting drag reduction effects have been developed (Meter, 1964; Kilbane and Greenkorn, 1966; Seyer and Metzner, 1967; Virk et al., 1967). The correlations for drag-reducing fluids, which have in part a theoretical reasoning, are those of Meyer (1966), Seyer and Metzner (1969a), and Astarita et al. (1969).

The approach of Seyer and Metzner (1969a) is the most popular and uses the logarithmic similarity laws for drag-reducing systems based on theoretical arguments similar to those of Millikan (1939) for Newtonian fluids and of Dodge and Metzner (1959) for purely viscous non-Newtonian fluids. The velocity profile for the turbulent core in smooth circular pipes proposed by Seyer and Metzner (1969a) has the following form

$$u^+ = A' \ln y^+ + B' \tag{1}$$

where

$$A' = 2.46 \tag{2}$$

$$B' = 5.6 + 1.55\, De \ (\text{for } 0 \leq De \leq 10) \tag{3}$$

The definitions of u^+ and y^+ are given in the notation; De is of the form given below

$$De = \frac{\theta_{fl} u^{*2}}{\nu} \tag{4}$$

Since drag-reducing fluids are known to be Newtonian in viscosity but exhibit mild elasticity, use is made of a dimensionless Deborah number (De) defined as the ratio of the fluid relaxation time (θ_{fl}) and characteristic process time (u^{*2}/ν).

During steady shear flow of polymer solutions, the molecules are known to undergo extensions. However, due to the elastic nature of the fluids, the molecules show a tendency to relax. The material constant θ_{fl} can be considered as representing the time required for molecular relaxation and would therefore be a measure of the degree of fluid elasticity.

There is considerable discussion in the literature (Astarita, 1965; Seyer and Metzner, 1969b; Kelkar and Mashelkar, 1972; Virk, 1975) wherein it has been explicitly demonstrated that choice of the characteristic time scales of the above kind for defining De is completely adequate for correlating the frictional characteristics in drag reducing flows. Reported experimental studies on the determination of fluid relaxation times (θ_{fl}) published in the literature (Seyer and Metzner, 1969b) show that θ_{fl} varies as $\dot{\gamma}^{-m}$, where m lies between 0.5 and 1.0. The general practice is to assume m as equal to 1 so that Deborah number can be taken as a constant independent of shear rate knowing that u^{*2}/ν is proportional to wall shear rate.

From the velocity profile given by Eq. 1, the relationship between friction factor and Reynolds number could be easily derived by determining the average velocity by integration and assuming a linear velocity profile in the viscous sublayer to yield

$$\sqrt{\frac{2}{f}} = A'(1-\xi_l)^2 \ln \frac{Re\sqrt{f}}{2\sqrt{2}} + (1-\xi_l)^2 - G \tag{5}$$

Equation 5 was found by Seyer and Metzner (1969a) to fit available friction factor data for drag-reducing fluids within a mean deviation of 4.2% for a value of $G = 3.0$. Despite the excellent agreement between the correlating equations and experimental data, the velocity profile given by Eq. 1 has an inherent incongruity in failing to predict a zero velocity gradient at the centerline. Such an incongruity has existed in all the earlier suggested velocity profiles both for Newtonian fluid (e.g., Nikuradse, 1932) and inelastic non-Newtonian fluids (Dodge and Metzner, 1959; Clapp, 1961; Bogue and Metzner, 1963). New velocity profile models for turbulent flow in smooth pipes were suggested by Stein et al. (1980) for Newtonian fluids and by Shenoy and Saini (1982) for inelastic non-Newtonian power law fluids by proper adjustment of the parameters so that the centerline velocity gradient was zero. The same approach can be extended to mildly elastic drag-reducing fluids so that a new velocity profile, devoid of this limitation of the centerline velocity not being zero, can be evolved.

For drag-reducing fluids, we assume the velocity profile in the turbulent core to be of the form

$$u^+ = A\,[\ln y^+ + C(\xi, De)] + B(De) \tag{6}$$

where $C(\xi,De)$ is a correction function having the following form similar to that used earlier by Stein et al. (1980), as well as Shenoy and Saini (1982), based on the suggestion by Hinze (1955):

$$C(\xi,De) = \sigma_1(De)\exp\left\{-\frac{1}{2}\left[\frac{\xi-0.8}{\sigma_2(De)}\right]^2\right\} \tag{7}$$

In order that Eq. 6 should satisfy the condition that the velocity gradient becomes zero at the centerline and also give the form of the friction factor given by Eq. 5 for drag-reducing fluids when integrated, it is imperative that the correction function should satisfy the following condition:

$$C'(1,De) = -\frac{0.2\sigma_1(De)}{\sigma_2^2(De)}\exp\left[-\frac{0.02}{\sigma_2^2(De)}\right] = -1 \tag{8}$$

The following forms of $\sigma_1(De)$ and $\sigma_2(De)$ were seen to be adequate for all values of De from 0 to 10 in order to satisfy the above condition:

$$\sigma_1(De) = 0.4398 + 0.123\,De + 0.0135\,De^2 \tag{9}$$

$$\sigma_2(De) = 0.254\,(1 + 0.2\,De) \tag{10}$$

Note that for $De = 0$, the values of $\sigma_1(0)$ and $\sigma_2(0)$ are equivalent to those obtained by Stein et al. (1980) for Newtonian fluids.

Equation 6 now ought to be integrated over the entire cross section of the turbulent core, i.e., from the laminar sublayer to the centerline. Since the integration involves evaluation of complex error functions whose definitive values have to be determined, some of the factors were decided to be evaluated over the entire cross section of the pipe as a first approximation. Thus, using the definition of $y^+ = \xi Re\sqrt{f}/2\sqrt{2}$, an implicit expression for the friction factor can be derived in a manner similar to that of Stein et al. (1980) for the Newtonian case to yield the following:

$$\sqrt{\frac{2}{f}} = A(1-\xi_l)^2 \ln\frac{Re\sqrt{f}}{2\sqrt{2}} + (1-\xi_l)^2 B - AD \tag{11}$$

It is found that the values of D, obtained during the determination of the mean dimensionless velocity, incorporated terms obtained from the evaluation of the error functions and could be approximated by the following equation:

$$D = 1.3676\,(1 - 0.09\,De - 0.01\,De^2) \tag{12}$$

Comparison of Eqs. 5 and 11 yields the expressions for A and B for the first approximation of $\xi_l \approx 0$, which, when substituted in Eq. 6 along with the expression for C, gives the dimensionless velocity profile for the turbulent flow of mildly elastic drag-reducing fluids as

$$u^+ = 2.46\left[\ln y^+ + (0.4398 + 0.123\,De + 0.0135\,De^2)\exp\left[-\frac{(\xi-0.8)^2}{0.129(1+0.2\,De)^2}\right] + 1.3676(1 - 0.09\,De - 0.01\,De^2)\right] + 5.6 + 1.55\,De - G \tag{13}$$

Note than in Eq. 13, if G is taken equal to 3.0, as suggested by Seyer and Metzner (1969a), then an error of about 4% is seen during the comparison of Eqs. 1 and 13. However, a choice of $G = 4.0$ yields an excellent agreement within less than 1.0% error for all values of De over a range of Reynolds numbers from 10^4 to 10^6. The present model, of course, additionally predicts a zero velocity gradient at the centerline for all De values ($0 \leq De < 10$), while the expression of Seyer and Metzner (1969a) fails in this respect.

NOTATION

A, A'	= numerical constants in Eqs. 1 and 6
$B(De), B'$	= functions of De in Eqs. 1 and 6
$C(\xi,De)$	= correction function, defined by Eq. 7
$C'(1,De)$	= first derivative of correction function evaluated at the centerline, defined by Eq. 8
D	= diameter of pipe
$D(De)$	= function of De appearing in Eq. 11, defined by Eq. 12
De	= Deborah number, defined by Eq. 4
f	= friction factor
R	= radius of pipe
Re	= Reynolds number, defined as $DV\zeta/\mu$
u	= axial velocity in pipe
u^+	= dimensionless axial velocity in pipe, defined as u/u^*
V	= average velocity in pipe
y	= distance from the wall
y^+	= local friction Reynolds number, defined as $yu^*\zeta/\mu$
y_l^+	= local friction Reynolds number at the edge of the laminar sublayer

Greek Letters

$\dot{\gamma}$	= wall shear rate
θ_{fl}	= fluid relaxation time
ν	= kinematic viscosity
ξ	= dimensionless distance from the wall, defined as y/R
ξ_l	= dimensionless viscous sublayer thickness, defined as $y_l{}^+ 2\sqrt{2}/Re\sqrt{f}$
ζ	= fluid density
μ	= fluid viscosity
$\sigma_1(De)$	= function of De in Eq. 8, defined by Eq. 9
$\sigma_2(De)$	= function of De in Eq. 8, defined by Eq. 10

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ERRATA

• For the R&D note entitled "Analysis of Pressure Fluctuations in a Gas-Solid Fluidized Bed" (March, 1984) by L. T. Fan, S. Hiraoka, and S. H. Shin, the following corrections are needed:

p. 347, col. 2, line 17 should read: "$\sqrt{\overline{(P_B')^2}}$."

p. 347, col. 2, line 21 should read:

$$\text{“}\sqrt{\overline{(P_B')^2}} = \left(\frac{M_s}{2A}\right)(\overline{U}_0 - U_{mf})\frac{1}{\sqrt{2\pi}}.\text{”}$$

p. 347, col. 2, line 34 should read: "$\theta = s\overline{T}$."

p. 348, col. 1, lines 1 and 2 should read:

$$\text{“}-\omega^2(1 - \cos\omega\overline{T}) + \left(\frac{2\overline{P}_cA^2}{v_cM_s}\right) = 0 \quad (27)$$

$$-\omega\,\sin\omega\overline{T} + \left(\frac{2n\overline{U}_0^{n-1}K_DA}{M_s}\right) = 0 \quad (28)\text{”}$$

p. 348, col. 2, line 6 should read: "fluidized-bed fluctuations, the dimensionless parameters, θ' and."

p. 348, col. 2, lines 8 and 9, should read:

$$\text{“}\theta' = \omega\overline{T} = 2\pi f\frac{L_{mf}}{\overline{U}_{br}}$$

$$\xi^2 = \frac{2\overline{P}_cA^2}{v_cM_s}\overline{T}^2 = \frac{2\overline{P}_cL_{mf}}{(v_c/A)\rho_{mf}\overline{U}_{br}^2} \quad (35)\text{”}$$

p. 348, col. 2, lines 11 and 12 should read:

$$\text{“}\theta'^2(1 - \cos\theta') - \xi^2 = 0 \quad (36)$$

$$2m\pi < \theta' < (2m + 1)\pi,\ m = 0, 1, 2, \ldots \quad (37)\text{”}$$

p. 348, col. 2, line 30 should read: "$\theta' < \pi$ while small particles do in the range of $2\pi < \theta' < 3\pi$. Figure."

p. 349, col. 2, line 25 should read: " $'$ = fluctuating component."

• For the "Optimum Pore Size for the Catalytic Conversion of Large Molecules" by E. Ruckenstein and M. C. Tsai (May, 1981): "The effective diffusion coefficient D_{eff} which includes the partition coefficient K_p" should be replaced in Eq. 6 by "the diffusion coefficient D_{eff}/K_p." This changes Eq. 15 to $\lambda_{opt} = 0.46$.